35.7 A $1.5in$ diameter steel shaft $(E = 2.9 \times 10^7 psi)$ is $4ft$ long and supported by two frictionless bearings at its ends. A $200lb_m$ flywheel is mounted on the center of the shaft. The shaft weight is negligible and there is no damping. What is the critical speed of the shaft?

A. $4Hz$

B. $12Hz$

C. $78Hz$

D. $590Hz$

The critical speed depends on the linear natural frequency of the shaft which can be determined from the static deflection due to the mass of the flywheel modeled as a point load applied to the center of a simple beam.

Start by calculating the area moment of inertia for a the shaft.

$$I = \frac{\pi r^4}{4} = \frac{\pi\,(0.75in)^4}{4} = 0.2485in^4$$

Find the formula for the static deflection of a simple beam with a concentrated load at center. Calculate the maximum static deflection.

$$\delta_{st} = y = \frac{Pl^3}{48EI} = \frac{(200lb_f)\,(48in)^3}{48\left(2.9 \times 10^7 \frac{lb_f}{in^2}\right)(0.2485in^4)} = 0.064in$$

The undamped natural circular frequency can then be determined as a function of the static deflection.

$$\omega_n = \sqrt{\frac{g}{\delta_{st}}} = \sqrt{\frac{\left(32.2\frac{ft}{s^2}\right)\left(\frac{12in}{1ft}\right)}{0.064in}} = 77.7\frac{rad}{s}$$

Find the linear natural frequency.

$$f_n = \frac{\omega_n}{2\pi} = \frac{77.7\frac{rad}{s}}{2\pi} = 12.4Hz$$

Answer B